$$\Delta x(t) = \Delta x_0 + \frac{v_0}{2}\frac{\mu}{(1-\nu)\sigma_0}\beta_-^P t.$$

As is evident, $\Delta x(t)$ increases in proportion to the time t and the amplitude of the plastic wave β_-^P.

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NUMERICAL ANALYSIS OF THE NONLINEAR STABILITY OF VIBRATIONS IN A PLATE LYING ON A LAYER OF VISCOUS, COMPRESSIBLE LIQUID

V. N. Belonenko, O. Yu. Dinartsev, and
A. B. Mosolov

UDC 532.5

Problems related to the stability of vibrations of mechanical systems that are in contact with a viscous, compressible liquid often arise in many areas of science, engineering, and contemporary production. A typical example is the problem concerning the stability of heavily loaded friction nodes under conditions of increased vibration.

In order to take into account the compressibility of liquid described by the Newtonian model with linear viscosity, one must consider both the shearing viscous stresses and the volumetric viscous stresses (which is usually not the case) [1]. The assumption that the coefficient of volumetric viscosity is zero is in most cases unjustified, and for some liquids the coefficient of volumetric viscosity can be many times (sometimes many orders) greater than the coefficient of ordinary shearing viscosity. Also, when the forces that act on a liquid are intense, one cannot ignore the dissipation of energy for a change in volume. For vibrational processes that are accompanied by a change in volume, the effect of volumetric viscosity can be very substantial.

1. Formulation of the Problem and a Determination of Equations. We will consider the one-dimensional problem of forced vibrations in a massive layer S that lies on a layer of viscous compressible liquid (Fig. 1) acted on by a periodic force F(t).

The basic equations of the problem are:

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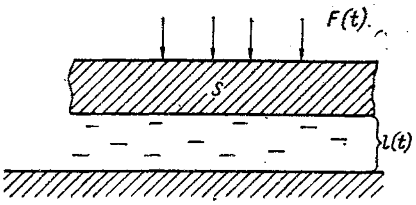

Fig. 1

the equation of motion

$$ml_{,tt} = p - \left(\eta_V + \frac{4}{3}\eta_s\right)\frac{l_{,t}}{l} - mg + F(t); \quad (1.1)$$

and the equation of continuity (conservation of mass of the liquid)

$$\rho l = \rho_0 l_0. \quad (1.2)$$

Here, m is the mass per unit length of the layer S; $\ell(t)$, ℓ_0 are the current and initial thicknesses of the gap; $\rho(t)$, ρ_0 are the current and initial densities of the liquid; p is the pressure in the liquid layer; η_V and η_S are the coefficients of volumetric and shearing viscosity; and g is the acceleration due to gravity.

If there is a discontinuity in the solidity (a break between layer S and the liquid layer), then Eq. (1.2) is no longer satisfied.

The force is

$$F(t) = \sigma_1 \sin\omega t - \sigma_2 - p_0, \ \sigma_2 = \sigma_1 - mg, \quad (1.3)$$

where p_0 is the atmospheric pressure; $\omega = 2\pi\nu$; ν is the frequency of the force. It is assumed that $4\ell_0\nu c^{-1} \ll 1$, where c is the velocity of sound in the liquid.

For high pressures (under isothermic conditions), many liquids will satisfy the Tait equation of state [2, 3]

$$P(\rho) = (p_0 + B)\left\{\exp\left[\frac{1}{A}\left(1 - \frac{\rho_0}{\rho}\right)\right] - 1\right\} + p_0. \quad (1.4)$$

This dependence is also used in our study, and it is assumed that layer S may lose contact with the liquid during motion (this occurs when the pressure in the liquid goes to zero). It then follows that the pressure p in Eq. (1.1) must take the form

$$p = P(\tilde{\rho})h(P(\tilde{\rho})), \ \tilde{\rho} = l_0\rho_0/l, \quad (1.5)$$

where h is the Heaviside function.

The coefficient of shearing viscosity as a function of the pressure behaves in correspondence with the Barus law [4], and a similar expression is used for the coefficient of volumetric viscosity [5]

$$\eta_s = \eta_{s_0}\exp(\alpha_s\Delta p)\,h(P), \quad \Delta p = p - p_0, \quad \eta_V = \eta_{V_0}\exp(\alpha_V\Delta p)\,h(P). \quad (1.6)$$

Equations (1.1)-(1.6) fully describe the problem of vibrations of a layer S and can be directly used for a numerical analysis of the stability of motion. However, for many liquids, such an analysis cannot be done because of the lack of data on the coefficients of viscosity and compressibility as functions of the state parameters. Liquid lubricants have been the most thoroughly analyzed in this respect, and, therefore, we will use data for a certain type of machine oil (AU spindle oil at 40°C) [3, 5]: ρ_0 = 894.3 $Kg\cdot m^{-3}$, $\alpha_V \cong \alpha_S$ = 0.02018 MPa^{-1}, A = 0.08264, B = 128 MPa, c = 1388.8 $m\cdot sec^{-1}$, η_{S_0} = 15.62 MPa·sec, $\eta_{V_0} = \zeta\eta_{S_0}$. It is assumed that p_0 = 0.1 MPa, $\ell_0 m$ = 100 $Kg\cdot m^{-1}$, ν = 1 kHz.

The quantity of the volumetric viscosity strongly depends on the actual chemical composition of the oil. Introduction of the corresponding impurities in [6] can change ζ by several orders without changing the remaining parameters. Therefore, it is natural to consider the dimensionless quantity ζ as a variable parameter and to investigate its effect on the stability of the vibrations of the layer.

2. Analysis Method. Assuming that $x = \ell_0^{-1}\ell$, $\tau = \omega t$, $y = \dot{x}$, $\theta \equiv \tau$, we will rewrite Eqs. (1.1), (1.2) [taking into account (1.3)-(1.6)] in the form of a dimensionless, autonomous system in 2π-periodic phase space $(x, y, \theta) \in R_+^2 \times S^1$, $R_+^2 = \{(x, y) \in R^2 \mid x \geqslant 0\}$:

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}), \quad \mathbf{X} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}, \quad \mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}, \tag{2.1}$$

where $f_1(x, y, \theta) \equiv y$; $f_2(x, y, \theta) = \alpha\varphi(x)h(\varphi(x)) - \left(\zeta + \frac{4}{3}\right) byx^{-1} \exp\left[\gamma\left(\varphi(x) - \frac{\beta}{\alpha}\right)\right] h(\varphi(x)) + d\sin\theta - \beta$; $f_3(x, y, \theta) \equiv 1$; $\alpha = (p_0 + B)q$; $\beta = p_0 q$; $b = \eta_{s_0}\omega q$; $\gamma = (B + p_0)\alpha_V$; $d = \sigma_1 q$; $q = (ml_0\omega^2)^{-1}$; the dot denotes derivation with respect to τ; and $\phi(x)$ is the dimensionless pressure.

For an analysis of system (2.1), we will use the Poincaré representation method, which allows one to obtain a visual representation for the dynamics of the system. For a determination of the Poincaré representation, we will consider the two-dimensional section Γ of the three-dimensional phase space of the system. For example, one can select $\Gamma \equiv \Gamma_0 = \{(x, y, \theta) \mid \theta = 0\}$. The Poincare representation P: $\Gamma_0 \to \Gamma_0$ is created by the flow ψ_t: $R_+^2 \times S^1 \to R_+^2 \times S^1$ given by (2.1) and is determined in the following way

$$P(x, y) = \pi \circ \psi_{2\pi}(x, y, 0),$$

where π is the projection onto R_+^2. It is easy to show that the solution of (2.1) is restricted for $t \geq 0$, and, therefore, the Poincare representation is determined globally.

For dissipative systems it is known that the phase volume is compressed for motion. The rate of compression is $\lambda = \operatorname{div}\mathbf{f}$. If λ is constant, then one observes uniform, homogeneous compression of the entire phase space. This case has been studied in detail in literature.

For system (2.1)

$$\lambda = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial \theta} = -\left(\zeta + \frac{4}{3}\right) bx^{-1} \exp\left[\gamma\left(\varphi(x) - \frac{\beta}{\alpha}\right)\right] h(\varphi(x))$$

is a function of x and can go to zero when the layer S breaks away from the liquid and the system becomes Hamiltonian. At this stage, the phase space, according to Liouville's theorem, does not change. Nevertheless, on the average, the phase space is compressed after a cycle, and the Poincare representation compresses the area by Γ_0. Hence, one can expect that after the transitional state, the established motion in the phase space will occur along the "surface of lowest dimensionality" [7], and the trajectory of the system is attracted to some attractor. Well-known examples of the Lorentz model, the Duffing equation, or the nonlinear oscillator [7-11] indicate that the attractors are are extremely complicated. In the simplest case, they can be various limiting cycles (simply a point for the Poincare representation).

We will specify the above. The point M is called a stable, fixed point (the simple attractor) of the Poincare representation of period n if

$$P^n(M) = M, \ \|DP^n(M)\| < 1,$$

where $P^n = \underbrace{PP\ldots P}_{n}$, a $\|\cdot\|$ is the matrix norm that is induced by the ordinary Euclidean norm in R^2.

3. Numerical Experiment. One cannot construct the Poincare representation without the solution to system (2.1). It is necessary to use numerical techniques for obtaining this solution. There is an exception for small vibrations, when

$$\sigma_1/(c^2\rho_0) \ll 1, \ x = 1 + \delta, \ |\delta| \ll 1.$$

For such conditions, system (2.1) is linearized and reduces to a single second-order equation

$$\ddot{\delta} = -\frac{\alpha}{A}\delta - \left(\zeta + \frac{4}{3}\right) b\dot{\delta} + d\sin\tau.$$

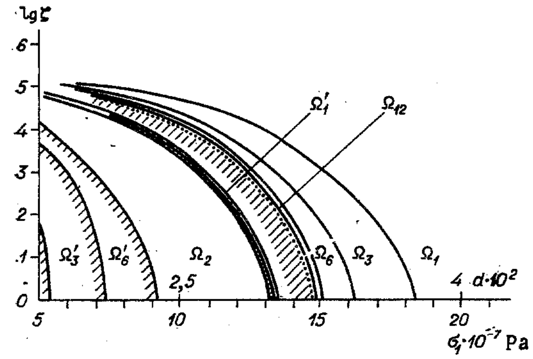

Fig. 2

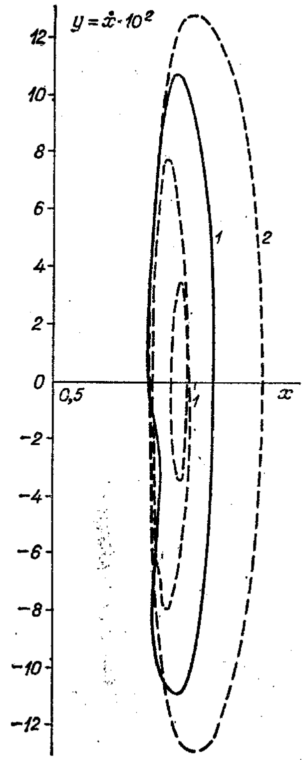

Fig. 3

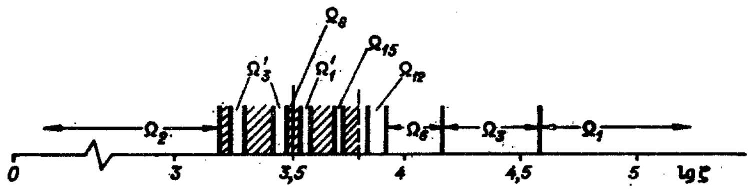

Fig. 4

Its solution has the form

$$\delta = k\sin(\tau + \varphi),$$

where

$$k = d\left[\left(1-\frac{\alpha}{A}\right)^2 + \left(\zeta+\frac{4}{3}\right)^2 b^2\right]^{-\frac{1}{2}};\quad \varphi = -\operatorname{arctg}\frac{\left(\zeta+\frac{4}{3}\right)b}{1-\frac{\alpha}{A}}.$$

Consequently, for the linearized case the Poincaré representation has a stable, fixed point of the period 1 $M = (1 + k\sin\phi,\ k\cos\phi)$.

For constructing the Poincaré representation, the complete system (2.1) is integrated on an ES-1033 computer using the Runge–Kutta method with the variables ζ and d. The range of change of $d = (1.25\text{–}12.25)\cdot 10^{-2}$ corresponds to a change in the amplitude of the loading over the limits σ_1 = 50-600 MPa, and the parameter ζ varies within the range $1\text{-}10^5$.

The calculation results are shown in Figs. 2-5. In Fig. 2, one can find the general form for the behavior of the system in the coordinates $\sigma_1(d) \sim \lg\zeta$, Ω_k is the range of change for the parameters that correspond to the periodic motion with a period k. It is interesting that in this case, in contrast to most cases considered in the literature on the subject, the loss in stability of the basic regular motion of period 1 (the range Ω_1) is related not to bifurcation of the doubling of the period but to the bifurcation of the tripling [11]. A further transition to chaotic motion occurs through a cascade of

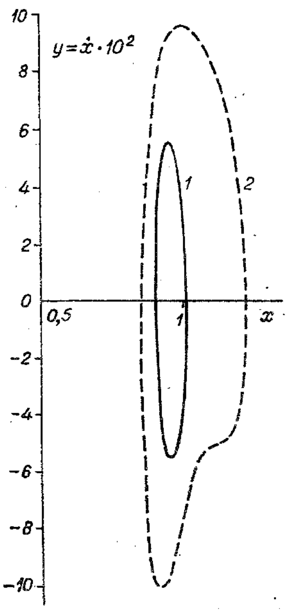

Fig. 5

successive bifurcations of the doubling of the period, which gives cycles with periods of 6, 12, 24, etc. The mathetical theory of these processes is given in [12], while physical and model examples of the effect of dissipation of nonlinearity on the behavior of autonomous, stochastic systems is studied in [13, 14]. The boundary of the region for periodic motion (the region of stability) is shown in Fig. 2 with the dotted line, while the dashed line marks the beginning of the region for chaotic motion.

On Fig. 3 in the phase plane $(x, y = \dot{x})$, the simplest attractor of period 1 is illustrated (the cycle $1 \in \Omega_1$, lg $\zeta = 2$, $\sigma_1 = 160$ MPa) along with the attractor with a period of 3 that is obtained after tripling of the period (cycle $2 \in \Omega_2$, lg $\zeta = 1$, $\sigma_1 = 160$ MPa).

Calculations indicate that the region of chaotic motion contains inside itself extensive regions of periodic motion, some of which are given in Fig. 2. A close analysis shows that it is all the more possible to distinguish the narrow zones of period motion inside the hatched regions. Their mutual positioning is shown in Fig. 4, while the structure of the "cut" is indicated in Fig. 2 for $\sigma_1 = 100$ MPa.

Figure 5 gives a representation of the behavior of the periodic solutions inside the "chaotic" region for $\sigma_1 = 90$ MPa. Cycles 1 ($\zeta = 7\cdot10^3$) for period 1 pertains to region $\Omega_1^{\prime}$, where cycle 2 ($\zeta = 6\cdot10^3$) of period 2 is taken from Ω_2. Comparing Figs. 3 and 5, it is easy to see the difference in the mechanism of change for the period of the motion. In the first case, the change in the period is related to the bifurcation of the tripling and is associated with the "separation" of the branches from the cycle of period 1, and in the second case, the periodic motion arises due to chaotic motion, and the motion is retarded in terms of the cycle without a change in topology.

The data presented above show that the stability of the layer (in the absence of chaos) significantly depends on the parameter ζ (at least in the range 50 MPa $< \sigma_1 <$ 190 MPa), i.e., really on the volumetric viscosity of the liquid. An increase in the volumetric viscosity on the whole has some effect on the stability of the system.

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STRENGTH EVALUATION FOR A WELDED JOINT WITH A THIN YIELDING INCLUSION OF SMALL SIZE

A. B. Borintsev, I. Yu. Devingtal',
Yu. A. Neoberdin, and A. V. Shvetsov

UDC 539.375

1. The strength of a welded joint depends on properties of the fusion zone, which may have the form of a thin layer with reduced strength and deformation properties with production defects, including inclusions (see, e.g., [1, 2]). The object of study in this work is a plane model of a welded joint (Fig. 1) which is two half-planes with elasticity moduli E_+ and E_-, and Poisson's ratio ν_+ and ν_- joined through a thin layer of thickness 2h; the E and ν of the layer material either conform with the corresponding elasticity constants of one of the welded materials, or they are intermediate between them (e.g., average). In a certain area the layer is interrupted by an extraneous, relatively yielding, thin inclusion with elasticity modulus E_0. In the Oxy coordinate system shown in Fig. 1 the inclusion occupies the region $|y| \leq h_0 g(x)$, where a is half the inclusion length, h_0 is half the average inclusion thickness ($h_0 \ll a$), and g(x) is a dimensionless shape function for the inclusion whose average value in the section from −a to +a equals unity, i.e., $[g(x)]_a = 1$.

Loading in the model being considered is accomplished at infinity with stress $\sigma_y^\infty = pf(x)$, where p is average stress in the section from −a to +a of axis x, and f(x) is a function of stress distribution inhomogeneity so that $[(f(x)]_a = 1$.

By a relatively yielding inclusion we understand one which leads to positive stress concentration at its ends in the thin layer. The thin layer simulates the fusion zone with reduced (compared with the materials being welded) mechanical properties. Therefore, sources for the start of failure are hypothetically assumed to be parts of the layer adjacent to the ends of the inclusion where there is an unfavorable combination of a high stress level with a low level of strength and deformation properties of the layer metal.

The aim of this work is determination of the critical value of applied load p for small inclusions which are often encountered in engineering practice, and estimation of their

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